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Compton scattering in a converging fluid flow – III Spherical supercritical accretion

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Summary. Radiative transfer in spherical, supercritical accretion on to a massive black hole is considered. Particular emphasis is placed on the case of non-adiabatic flow in which electron scattering in the converging flow is the dominant source of opacity and photon heating. In escaping diffusively, the photons, which are produced mainly at the trapping radius, undergo $\sim (c/u_{\rm tr})^2$ scatterings (where $u_{\rm tr}$ is the velocity of the flow at the trapping radius), each one giving a secular fractional energy increase $\sim (u_{\rm tr}/c)^2$ and a total average increase of order unity. The emitted radiation spectrum will be a superposition of the locally produced spectra at low frequencies and a power-law at high frequencies. For gas accreting radially with the free-fall speed, the spectral index observed in the outflowing radiation is $\alpha \sim 2$. Both thermal and non-thermal emission processes are discussed. The conditions under which bulk heating of the photons is important are specified.

1 Introduction

This is the third and final paper in a series describing Fermi acceleration of photons by a converging fluid flow. We apply the formalism developed by Blandford & Payne (1981a, Paper I) to the problem of spherical, supercritical accretion of ionized gas on to a black hole. In radial accretion, it is possible for the bulk velocity of the electrons to exceed the thermal velocity. Photons can then be preferentially accelerated by the converging flow of the accreting gas. If the gas, accreting at a rate $\dot{M}$, is in free-fall, then the electron-scattering optical depth to infinity from a radius r is

$$\tau_{\rm T}(r) = \frac{1}{2}\left(\frac{\dot{M}}{\dot{M}_{\rm E}}\right)\left(\frac{r}{r_{\rm s}}\right)^{-1/2}, \tag{1}$$

where $r_{\rm s}(\equiv 2GM/c^2)$ is the Schwarzschild radius and $\dot{M}_{\rm E}(\equiv 4\pi GMm_{\rm p}/\sigma_{\rm T}c)$ is the Eddington accretion rate. Hence, for supercritical accretion ($\dot{M} > \dot{M}_{\rm E}$), there exists a well-defined region of the flow for which $\tau_{\rm T}(r) > 1$ and from which photons must escape diffusively. This outward diffusion of the radiation is inhibited by its inward convection by the scattering electrons. The velocity of the inflowing electrons eventually becomes so large that photons are convected inwards more rapidly than they can diffuse outwards. The radius at which this occurs is the trapping radius r_{tr} (Rees 1978; Begelman 1979 and references therein), defined here by

$$\frac{u(r_{tr})}{c}\tau_T(r_{tr})\equiv\frac{1}{3} \tag{2}$$

(*cf.* equation 4), where u is the velocity of the flow. Most of the energy radiated to infinity is produced in the vicinity of the trapping radius. In escaping diffusively from r_{tr}, the photons undergo $\sim (c/u_{tr})^2$ scatterings $[u_{tr}\equiv u(r_{tr})]$, each one giving a secular fractional energy increase $\sim (u_{tr}/c)^2$ and a total average increase of order unity. The emitted radiation spectrum will be a superposition of locally produced spectra at low frequencies and have a power-law shape at high frequencies.

In many treatments of spherical accretion, the flow is assumed to be adiabatic, in which case the radiative efficiency is quite low (Shapiro 1973). However, if the flow is highly dissipative, perhaps due to the presence of turbulence or shock waves, then, as long as the cooling time-scale is short enough to ensure that the gas can radiate this energy on the inflow time-scale, spherical accretion can, in principle, be as efficient as disc accretion (Mészáros 1975; Rees 1978; McCray 1979).

In Section 2, we solve the problem of radiative transfer in a given spherical, supercritical flow when bulk acceleration dominates photon heating. Photon production and thermal effects for the case of bremsstrahlung emissivity are considered in Section 3, and for the case of synchrotron emissivity in Section 4. The relevance of this problem to the X-ray emission from quasars and active galactic nuclei is briefly discussed in Section 5.

2 Acceleration of photons in a spherical accretion flow

Consider first the problem of steady, spherically-symmetric accretion on to a black hole under conditions in which thermal Comptonization is ignorable. We compute the emitted spectrum for a δ-function source, by solving equation (18) of Paper I in this limit. We assume that the radial inward velocity $u\propto r^{-\beta}$ ($\beta=\text{constant}<2$) in the vicinity of the trapping radius. We will be most interested in flows in which radiation pressure and thermal pressure are ignorable and the trapping radius is well within the sonic radius. In this case, the gas is in free-fall with $\beta=1/2$.

As with the case of a radiation-dominated shock (Blandford & Payne 1981b, Paper II), there are three basic features associated with bulk acceleration in a converging fluid flow – convection, diffusion and the production of a power-law spectrum at high frequencies. The partial differential equation satisfied by the photon occupation number in a steady, radial flow is

$$\tau^*\frac{\partial^2 n}{\partial\tau^{*2}}+\left[\frac{d(\ln ur^2)}{d(\ln\tau^*)}-\tau^*\right]\frac{\partial n}{\partial\tau^*}+\frac{1}{3}\left[\frac{d(\ln ur^2)}{d(\ln\tau^*)}\right]\nu\frac{\partial n}{\partial\nu}=0, \tag{3}$$

where

$$\tau^*\equiv 3\frac{u(r)}{c}\tau_T(r)=\frac{3\dot{M}\sigma}{4\pi\bar{m}cr}, \tag{4}$$

$\bar{m}$ being the mean mass per scatterer and σ the scattering cross-section. (Note that the trapping radius corresponds to $\tau^*=1$.) We solve equation (3) with $u\propto r^{-\beta}$ by seeking separable solutions of the form

$$n(\nu,\tau^*)=f(\tau^*)\tau^{*3-\beta}\nu^{-\lambda}. \tag{5}$$

Here, λ is an eigenvalue of the confluent hypergeometric differential equation

$$\tau^* \frac{d^2 f}{d\tau^{*2}} + [(4-\beta)-\tau^*]\frac{df}{d\tau^*} - [(3-\beta) - \tfrac{1}{3}(2-\beta)\lambda] f = 0. \tag{6}$$

We are interested in the solution to equation (6) which corresponds to a constant spectral flux of photons as $\tau^* \to 0$ and adiabatic compression of the photons as $\tau^* \to \infty$. The relevant solution can be expressed as an infinite sum of generalized Laguerre polynomials $L_n^{3-\beta}(\tau^*)$ (with the normalization of Abramowitz & Stegun 1970) and the corresponding eigenvalues λ_n are given by

$$\lambda_n = \frac{3(n+3-\beta)}{(2-\beta)}; \qquad n = 0, 1, 2, \ldots. \tag{7}$$

Photons of frequency ν_0 created at τ_0^* are accelerated locally by the compression of the flow at a rate

$$\frac{1}{\nu}\frac{d\nu}{dt} = (2-\beta)\frac{4\pi\bar{m}c}{9\dot{M}\sigma} u_0 \tau_0^*, \tag{8}$$

and this acceleration continues everywhere (that is, at optical depths both greater than and less than τ_0^* a photon will have a frequency $\nu > \nu_0$). If the photon production rate at τ_0^* is $\dot{N}$ (photons s^{-1}), then

$$n(\nu_0, \tau^*) = \frac{\dot{N}\tau_0^{*3}\bar{m}^2 c^5}{6(2-\beta)\dot{M}^2\sigma^2 u_0 \nu_0^3}\,\delta(\tau^* - \tau_0^*). \tag{9}$$

The solution to equation (3) subject to our boundary conditions is then

$$n(\nu, \tau^*) = \frac{\dot{N}\tau_0^{*3}\bar{m}^2 c^5}{6(2-\beta)\dot{M}^2\sigma^2 u_0 \nu_0^3} \exp(-\tau_0^*)\, \tau^{*3-\beta} \sum_{n=0}^{\infty} \frac{\Gamma(n+1)}{\Gamma(n+4-\beta)} L_n^{3-\beta}(\tau_0^*) \times L_n^{3-\beta}(\tau^*)\,(\nu/\nu_0)^{-[(n+3-\beta)/(2-\beta)]}, \tag{10}$$

for $\nu \geqslant \nu_0$. Making use of the bilinear generating function for Laguerre polynomials from Erdélyi *et al.* (1953), we find

$$n(x, \tau^*) = \frac{\dot{N}\tau_0^{*3}\bar{m}^2 c^5}{6(2-\beta)\dot{M}^2\sigma^2 u_0 \nu_0^3} \exp(-\tau_0^*) \left[\frac{\tau^* x}{(\tau^*\tau_0^* x)^{1/2}}\right]^{(3-\beta)} \times \frac{\exp[-(x/1-x)(\tau^* + \tau_0^*)]}{(1-x)} I_{3-\beta}\left[\frac{2}{1-x}(\tau^*\tau_0^* x)^{1/2}\right], \tag{11}$$

where we have introduced

$$x = (\nu/\nu_0)^{-3/(2-\beta)}, \tag{12}$$

and $I_{3-\beta}(x)$ is the Bessel function of imaginary argument. From equation (19) of Paper I we find the spectral flux of photons to be

$$F(\nu, \tau^*) = \frac{18\dot{M}^2\sigma^2}{\bar{m}^2}\frac{u\nu^2}{\tau^{*2}}\left(\frac{\partial n}{\partial \tau^*} + \frac{1}{3}\nu\frac{\partial n}{\partial \nu}\right). \tag{13}$$

The emergent photon flux for a delta-function source of strength $\dot{N}$ is then

$$F(x, 0) = \frac{3\dot{N}}{(2-\beta)\Gamma(3-\beta)}\frac{\tau_0^{*3-\beta}}{\nu_0} \exp[-\tau_0^*/(1-x)]\frac{x^{(5-\beta)/3}}{(1-x)^{4-\beta}}. \tag{14}$$

As a check on our analysis, it is of interest to compute the total outgoing and ingoing photon fluxes

$$F(\tau) = \int_{\nu_0}^{\infty} F(\nu, \tau)\, d\nu$$

$$= \dot{N}\,\frac{\Gamma(3-\beta, \tau_0^*)}{\Gamma(3-\beta)}, \qquad \tau^* < \tau_0^* \tag{15a}$$

$$= \dot{N}\,\frac{\gamma(3-\beta, \tau_0^*)}{\Gamma(3-\beta)}, \qquad \tau^* > \tau_0^* \tag{15b}$$

where $\Gamma(3-\beta, \tau_0^*)$ and $\gamma(3-\beta, \tau_0^*)$ are incomplete gamma functions with the property that $\Gamma(3-\beta, \tau_0^*) + \gamma(3-\beta, \tau_0^*) = \Gamma(3-\beta)$. Note that the ingoing and outgoing photon fluxes are constant, as required (since the scattering conserves photons), and that the total photon flux is $\dot{N}$.

From equation (13) we find that the emergent spectral power is

$$L(x, 0) = \frac{3h\dot{N}}{(2-\beta)\Gamma(3-\beta)}\,\tau_0^{*3-\beta}\,\frac{x\exp[-\tau_0^*/(1-x)]}{(1-x)^{4-\beta}}. \tag{16}$$

Fig. 1 shows the emergent luminosity for various injection radii (for the case in which the gas is in free-fall, i.e. $\beta = 1/2$). The total luminosity can also be computed from equation (13). It is

$$L(\tau) = \frac{\Gamma[(1+\beta)/3]}{\Gamma(4-\beta)}\,L_0\tau_0^{*3-\beta}\exp(-\tau_0^*)\,U[(1+\beta)/3, 4-\beta, \tau_0^*]$$
$$\times\{(3-\beta)M[-(2-\beta)/3, 3-\beta, \tau^*] - \tfrac{1}{3}\tau^* M[(1+\beta)/3, 4-\beta, \tau^*]\}, \qquad \tau^* < \tau_0^* \tag{17a}$$

$$= \frac{\Gamma[(1+\beta)/3]}{\Gamma(4-\beta)}\,L_0\tau_0^{*3-\beta}\exp(-\tau_0^*)\,M[(1+\beta)/3, 4-\beta, \tau_0^*]$$
$$\times\{U[-(2-\beta)/3, 3-\beta, \tau^*] + \tfrac{1}{3}\tau^* U[(1+\beta)/3, 4-\beta, \tau^*]\}, \qquad \tau^* > \tau_0^* \tag{17b}$$

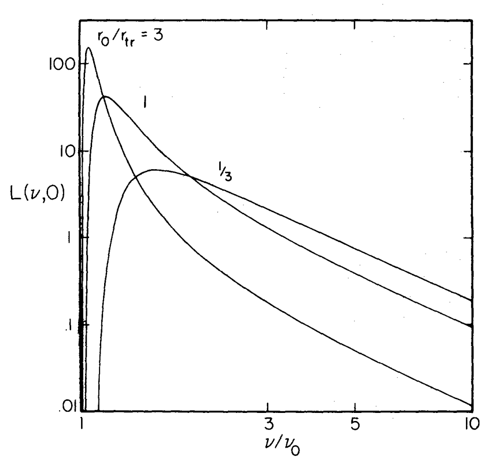

Figure 1. Emergent luminosity $L(\nu, 0)$ (in arbitrary units) for δ-function source of photons of frequency ν_0, plotted logarithmically. The curves correspond to different values of the injection radius r_0 (in units of the trapping radius r_{tr}). Note that the high frequency spectra are dominated by a power law. The results in this figure correspond to the case in which the gas is in free-fall.

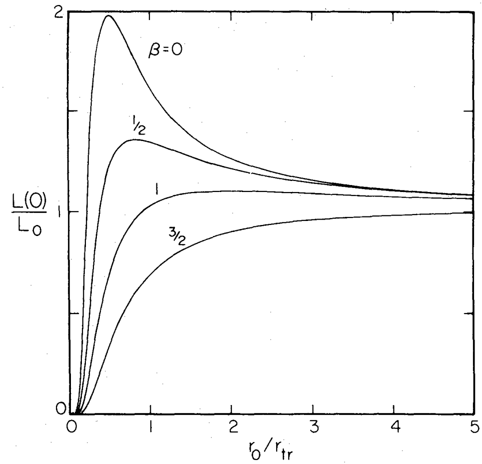

Figure 2. Energy amplification, $L(0)/L_0(L_0 \equiv h\nu_0 \dot{N})$, for emergent radiation as a function of the injection radius r_0 (in units of the trapping radius $r_{\rm tr}$), plotted linearly. The curves correspond to different values of the velocity profile $d \ln u/d \ln r = -\beta$(= constant).

where

$$L_0 \equiv h\nu_0 \dot{N} \tag{18}$$

is the source luminosity, and $M(a, b, x)$ and $U(a, b, x)$ are Kummer's functions (Abramowitz & Stegun 1970, ch. 13). Equations (17) for the luminosity differ from those of Begelman (1979) because our luminosity is that which would be measured by a stationary observer, whereas Begelman uses the luminosity which would be measured by an observer comoving with the fluid. From equation (17a), the total emergent luminosity is

$$L(0) = \frac{\Gamma[(1+\beta)/3]}{\Gamma(3-\beta)} L_0 \tau_0^{*3-\beta} \exp(-\tau_0^*)\, U[(1+\beta)/3, 4-\beta, \tau_0^*]. \tag{19}$$

For the case of free-fall, equation (19) simplifies to

$$L(0) = L_0[1 + 4/3\, \tau_0^*(1+\tau_0^*)] \exp(-\tau_0^*). \tag{20}$$

The maximum energy-amplification is then 1.36 and occurs for $\tau_0^* = 1.21$. In Fig. 2 we plot the energy amplification as a function of injection radius for various velocity profiles. We emphasize that in general we expect the gas to be in free-fall ($\beta = 1/2$), but Fig. 2 illustrates the general feature of photon energy amplification in a converging flow.

For a locally produced δ-function spectrum, or any sufficiently peaked spectrum, the transmitted flux will have a power-law shape at high frequency. If we define the spectral index as

$$\alpha \equiv -\frac{d[\log L(\nu, 0)]}{d(\log \nu)}, \tag{21}$$

then from equation (16) we obtain

$$\lim_{\nu \to \infty} \alpha = \frac{3}{(2-\beta)}. \tag{22}$$

For free-fall, $\alpha \to 2$ for $\nu \gg \nu_0$. Associated with this power-law is an increase in the mean photon energy.

3 Bremsstrahlung emission

3.1 EMITTED SPECTRUM INCLUDING PHOTON PRODUCTION

As a first example of the emitted spectrum when photon production is important, we consider the case of bremsstrahlung emission. Replacing $\dot{N}$ in equation (16) by $4\pi r_0^2\,\mathcal{N}(\nu_0, r_0)\,d\nu_0\,dr_0$, where the free–free photon emissivity is given by

$$\mathcal{N}(\nu, r) \propto n_e(r)^2\,T(r)^{-1/2}\,\frac{\exp\left[-h\nu/kT(r)\right]}{\nu}, \tag{23}$$

and integrating over $d\nu_0\,dr_0$ we obtain

$$L(\nu, 0) \propto \int_0^\infty \int_0^1 dx\,dt\,T(t)^{-1/2}\,\frac{t^{3(1-\beta)}}{(1-x)^{4-\beta}}\exp\left(-\frac{h\nu}{kT(t)}x^{(2-\beta)/3} - \frac{t}{(1-x)}\right). \tag{24}$$

We assume the temperature has a power-law profile,

$$T(\tau^*) = T_{tr}\,\tau^{*n}, \qquad n > 0 \tag{25}$$

so $T = T_{tr}$ at the trapping radius. To evaluate equation (24), note that the dominant contribution to the integral over t arises near the maximum in the exponential. The t-integral can therefore be evaluated by the method of steepest descents. The integral over x is then evaluated in the two limiting cases, $h\nu/kT_{tr} \ll 1$ and $h\nu/kT_{tr} \gg 1$. The results are,

$$L(\nu, 0) \propto (h\nu/kT_{tr})^{-[1/2 + (2\beta-1)/n]}, \qquad h\nu \ll kT_{tr} \tag{26a}$$

$$\propto (h\nu/kT_{tr})^{-3/(2-\beta)}, \qquad h\nu \gg kT_{tr} \tag{26b}$$

with $\beta < 2$. For $h\nu \gg kT_{tr}$ the $\nu^{-3/(2-\beta)}$ power-law arising from the converging fluid flow is recovered. For $h\nu \ll kT_{tr}$ we obtain another power-law, from the superposition of free–free spectra. For free-fall ($\beta = 1/2$), a $\nu^{-1/2}$ spectrum is obtained, arising from the $T^{-1/2}$ temperature dependence of free–free emission. It is obtained when gas radiating by this process is being accreted in free-fall (*cf.* Mészáros & Silk 1977). In Fig. 3 we plot the calculated emergent luminosity for the case of free–free emission from gas in free-fall, obtained by numerical integration of equation (24).

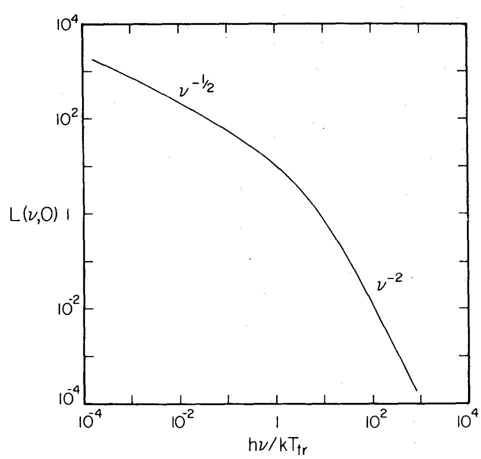

Figure 3. Emergent luminosity $L(\nu, 0)$ (in arbitrary units) under conditions in which the dominant photon production mechanism is bremsstrahlung emission from gas in free-fall, plotted logarithmically. The power-law at low frequencies arises from the superposition of free–free spectra, while the high-frequency spectrum is dominated by the power-law response to a δ-function source.

3.2 THERMAL BALANCE OF THE GAS

We now consider the thermal properties of the accreting gas radiating by the free–free process and assumed to be in free-fall. It is straightforward to generalize our results to other velocity fields. We postulate the existence of some internal dissipation within the supersonic flow (turbulence, shocks, etc.) capable of maintaining the temperature at $T(r)$.

It is useful to write the luminosity L in units of the Eddington luminosity $L_E(\equiv 4\pi GMm_pc/\sigma_T)$, $l \equiv L/L_E$ and the accretion rate $\dot{M}$ in units of the Eddington accretion rate $\dot{M}_E(\equiv L_E/c^2)$, $\dot{m} \equiv \dot{M}/\dot{M}_E$. Then the trapping radius is given by

$$r_{tr} = \frac{3}{2}\dot{m}r_s. \tag{27}$$

The photospheric radius (the radius at which the electron scattering optical depth $\tau_T = 1$) is

$$r_p = \frac{1}{4}\dot{m}^2 r_s. \tag{28}$$

If radiation is produced mainly by free–free emission near the trapping radius and escapes diffusively, its energy density is

$$\epsilon = l\left(\frac{L_E}{4\pi r_s^2 c}\right)\left(\frac{r}{r_s}\right)^{-2}, \quad r \gtrsim r_p \tag{29}$$

$$= \frac{1}{2}l\dot{m}\left(\frac{L_E}{4\pi r_s^2 c}\right)\left(\frac{r}{r_s}\right)^{-5/2}, \quad r_{tr} \lesssim r \lesssim r_p \tag{30}$$

and

$$l = 3\,(2/3\pi)^{1/2}\, f\dot{m}^2\left(\frac{\alpha m_e}{m_p}\right)\left(\frac{kT_{tr}}{m_ec^2}\right)^{1/2}\bar{g}. \tag{31}$$

In equation (31), $\bar{g}$ is the frequency-averaged Gaunt factor ($\bar{g} \sim 1.2$ for typical parameters) and

$$f \equiv \frac{\langle n_e^2\rangle}{\langle n_e\rangle^2}\,(>1) \tag{32}$$

is a factor which allows for clumpiness in the accreting gas. (The clumpiness may be tied in with the way the gas is supplied to the black hole, e.g. via tidal disruption or collisions of stars.) For simplicity we assume that f is constant and has values in the range $1–10^3$. If we view clumpiness in the flow as being the accretion of clouds of gas, then there must be a sufficient number of clouds to ensure that photons will diffuse out of the atmosphere and not escape freely. A sufficient condition for this is that the individual clouds be optically thin to scattering.

The thermal balance in the accreting gas depends on three time-scales: the infall time-scale, $t_i^{-1} \equiv u(r)/r$; the free–free cooling time-scale for the gas, $t_{b,g}^{-1} \equiv 2/3(2/3\pi)^{1/2}\alpha f n_e \sigma_T c\,(kT/m_ec^2)^{-1/2}\bar{g}$; and the Comptonization time-scale for the gas, $t_{c,g}^{-1} \equiv 4/3\,\epsilon\sigma_T c/m_ec^2$. Comparison of these time-scales leads to three inequalities. At the trapping radius

$$\left(\frac{kT}{m_ec^2}\right) < \frac{1}{4\pi}(2/3)^3(\alpha f\dot{m}\bar{g})^2 \Rightarrow t_{b,g} < t_i, \tag{33}$$

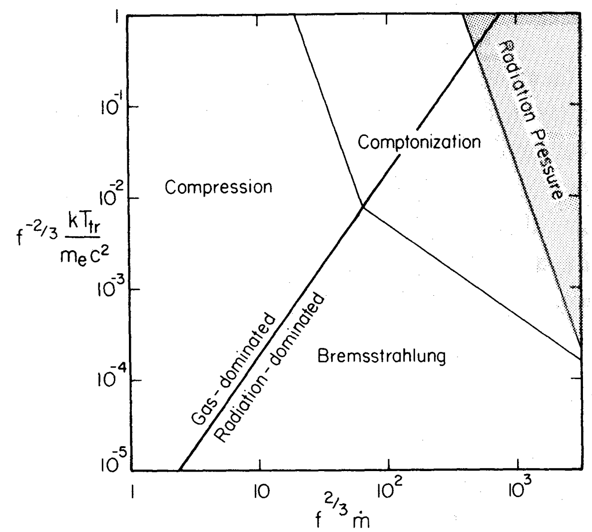

Figure 4. Thermal state of the gas (assumed to be in free-fall) at the trapping radius as a function of the temperature at the trapping radius T_{tr}, the clumpiness factor $f(\equiv\langle n_e^2\rangle/\langle n_e\rangle^2$, assumed constant), and the accretion rate $\dot{m}(\equiv\dot{M}/\dot{M}_E)$. Each sector is labelled to indicate the shortest time-scale: bremsstrahlung, Compton or infall (compressional regime). To the left of the bold line the plasma is gas-dominated, while to the right it is radiation-dominated. In the shaded region, radiation pressure influences the dynamics of the flow and our calculations are no longer valid.

$$\left(\frac{kT}{m_e c^2}\right) < \frac{1}{2}\dot{m}^{-1} \Rightarrow t_{b,g} < t_{c,g}, \tag{34}$$

$$\left(\frac{kT}{m_e c^2}\right) < \pi(3/2)^3(\alpha f \dot{m}^2 \bar{g})^{-2} \Rightarrow t_i < t_{c,g}. \tag{35}$$

For radii $r > r_{tr}$, $t_{b,g} \propto r^{3/2}T(r)^{1/2}$, $t_i \propto r^{3/2}$ and $t_{c,g} \propto r^{5/2}$, with $T(r)$ a decreasing function of increasing r. For a given accretion rate and temperature at the trapping radius we are interested in the shortest time-scale. There are thus three distinct regimes of thermal balance (the three sectors of Fig. 4).

(a) *Bremsstrahlung regime.* Since $T(r)$ is a decreasing function of increasing r, we expect that, if $t_{b,g}$ is the shortest time-scale at the trapping radius, it will almost certainly be so for $r > r_{tr}$. For adiabatic flow the only source of heat is the compression of the gas. However, since $t_{b,g} < t_i$, radiative processes are always capable of cooling the gas on the infall time-scale and the temperature remains very low. For non-adiabatic flows though, turbulence is capable of depositing considerably more energy into the gas. The electron temperature for $r \gtrsim r_{tr}$ is then fixed by a balance between turbulent heating and bremsstrahlung cooling. It is of interest to consider whether the thermal balance of the gas can be influenced by Compton heating. If $T \propto r^{-n}$ with $0 \leqslant n \leqslant 2$, then Compton heating is unimportant everywhere within the photosphere if it is unimportant at the trapping radius. If $n > 2$, then Compton heating may become important at some radius, in which case the gas will be heated until free–free cooling can balance Compton heating. Under certain conditions then, Compton heating may act to regulate the temperature gradient in the flow. For Compton heating to be unimportant at the photosphere, we require that the temperature at the photosphere $T_p > (4kT_{tr}/m_e c^2)^2 T_{tr}$. With $0 \leqslant n \leqslant 1$, Compton heating is unimportant beyond the photosphere if it is unimportant at the photosphere; while for $n > 1$ Compton heating may eventually become important, again regulating the temperature gradient.

The maximum temperature attainable at the trapping radius in the bremsstrahlung regime is

$$\frac{kT_{\mathrm{tr,\,max}}}{m_{\mathrm{e}}c^2} = \frac{1}{3}(2\pi)^{-1/3}(\alpha f\bar{g})^{2/3} \simeq 7.7\times10^{-3} f^{2/3}. \tag{36}$$

For example, with a clumping factor $f \sim 100$, $T_{\mathrm{tr,\,max}} \sim 10^9$ K.

(b) *Comptonization regime.* Here the thermal structure is fixed by a balance between Compton heating and Compton cooling, so the flow is roughly isothermal with temperature

$$T(r) = \tfrac{1}{4} h\,\langle\nu\rangle/k, \quad = T_{\mathrm{tr}}. \tag{37}$$

The maximum temperature attainable in this regime is $T_{\mathrm{tr,\,max}} \sim m_{\mathrm{e}}c^2/k$, limited mainly by electron–positron pair production.

The Comptonization time-scale increases with radius more rapidly than the bremsstrahlung time-scale, hence at some radius $r > r_{\mathrm{tr}}$, the flow may pass into the bremsstrahlung regime, in which case the discussion of Section (a) applies.

(c) *Compressional regime.* In this regime neither free–free nor Compton cooling can balance the compressional heating. If the flow were roughly adiabatic then the temperature profile in this regime would be $T(r) \propto r^{-1}$, while the presence of turbulence could steepen the temperature profile. The maximum temperature attainable in this regime is again $T_{\mathrm{tr,\,max}} \sim m_{\mathrm{e}}c^2/k$.

The time-scale for compressional heating increases with radius more rapidly than the bremsstrahlung time-scale, hence at some radius $r > r_{\mathrm{tr}}$, the flow may pass into the bremsstrahlung regime in Section (a).

Our discussion of the thermal balance in the inflowing gas is only valid when bremsstrahlung is the dominant mechanism for producing photons. At temperatures below $\sim 10^7$ K, bound–free and bound–bound processes dominate and their contributions must be determined self-consistently under the conditions of high radiation-density present in our model.

Also of interest in understanding the thermal structure of the flow is the Mach number, $\mathcal{M}$. The flow is radiation-dominated at the trapping radius when

$$\dot{m} > 3(3\pi/2)^{1/2}(\alpha f\bar{g})^{-1}\left(\frac{kT_{\mathrm{tr}}}{m_{\mathrm{e}}c^2}\right)^{1/2}, \tag{38}$$

and gas-dominated when the inequality is reversed.

Inside the photospheric radius

$$\mathcal{M}_{\mathrm{r}}^2 = \tfrac{1}{2} l^{-1}, \tag{39a}$$

$$\mathcal{M}_{\mathrm{g}}^2 = \tfrac{1}{5}\left(\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}}\right)\left[\dot{m}\left(\frac{kT}{m_{\mathrm{e}}c^2}\right)\left(\frac{r}{r_{\mathrm{tr}}}\right)\right]^{-1}, \tag{39b}$$

where the subscripts r and g denote radiation-dominated and gas-dominated flows respectively. l is given by equation (31) for the case of bremsstrahlung emission. Note that, for radiation-dominated flows, $\mathcal{M}$ is approximately constant. For cases of interest in this paper, $\mathcal{M} \lesssim 10$ within the photosphere for radiation-dominated flows. Outside the photospheric radius we find analogous results

$$\mathcal{M}_r^2 = \tfrac{1}{2}\, l^{-1} (r/r_p)^{-1/2}, \tag{40a}$$

$$\mathcal{M}_g^2 = \tfrac{6}{5}\left(\frac{m_p}{m_e}\right)\left[\dot{m}^2\left(\frac{kT}{m_e c^2}\right)\left(\frac{r}{r_p}\right)\right]^{-1}. \tag{40b}$$

3.3 THE RADIATION FIELD

The properties of the radiation field depend on the infall time-scale, the free–free cooling time-scale for the radiation field, $t_{b,r}^{-1} \equiv 2(2/3\pi)^{1/2}\alpha f n_e^2 \sigma_T c (kT/m_e c^2)^{1/2}(m_e c^2/\epsilon)\bar{g}$, and the Comptonization time-scale for the radiation field, $t_{c,r}^{-1} \equiv (4kT/m_e c^2) n_e \sigma_T c$. The Comptonization and free–free cooling time-scales for the matter and radiation differ because the two fluids have different heat capacities. $t_{b,r} < t_{c,r}$ is ensured when

$$\left(\frac{kT_{tr}}{m_e c^2}\right) < \tfrac{1}{2}\,\dot{m}^{-1} \tag{41}$$

(*cf.* equation 34). The same condition guarantees that bulk heating by the converging fluid flow is more important at r_{tr} than Comptonization by the hot electrons, since for the radiation field $t_{b,r} = t_i$ at r_{tr}.

When bulk heating and bremsstrahlung cooling are more important than Comptonization, the emitted radiation spectrum will be a superposition of free–free spectra below

$$\nu_{min} \sim 3kT_{tr}/h. \tag{42}$$

Above ν_{min} it will be a power-law with index $\alpha \sim 2$ extending to

$$\nu_{max} \sim \tfrac{1}{3} m_e u_{tr}^2/h, \tag{43}$$

determined by balancing the bulk acceleration rate with the rate of loss of energy by Compton recoil. For $\nu_{min} \lesssim \nu \lesssim \nu_{max}$, Compton effects will then be less important than bulk acceleration (*cf.* equations 28–30 of Paper I). So

$$\frac{h\nu_{max}}{m_e c^2} \sim \tfrac{2}{9}\,\dot{m}^{-1}, \tag{44}$$

and

$$\frac{\nu_{max}}{\nu_{min}} \sim \tfrac{2}{27}\,\dot{m}^{-1}\left(\frac{kT_{tr}}{m_e c^2}\right)^{-1}. \tag{45}$$

We have ignored the dynamical effects of the outflowing radiation on the velocity field. This is valid as long as $l \lesssim 1$. For bremsstrahlung emission this requirement implies

$$\dot{m} < \left(\frac{\pi}{6}\right)^{1/4}\left(\frac{\alpha f m_e \bar{g}}{m_p}\right)^{-1/2}\left(\frac{kT_{tr}}{m_e c^2}\right)^{-1/4}. \tag{46}$$

In Fig. 5 we have plotted the inequality which must be satisfied for neglect of Comptonization, equation (41), various values of l and (ν_{max}/ν_{min}), and the photon to electron ratio, given by

$$\frac{n_\phi}{n_e} = (6\pi)^{-1/2}\alpha f \dot{m}^2\left(\frac{kT_{tr}}{m_e c^2}\right)^{-1/2}\left(\frac{r}{r_s}\right)^{-1}\bar{g}, \tag{47}$$

all evaluated at r_{tr}.

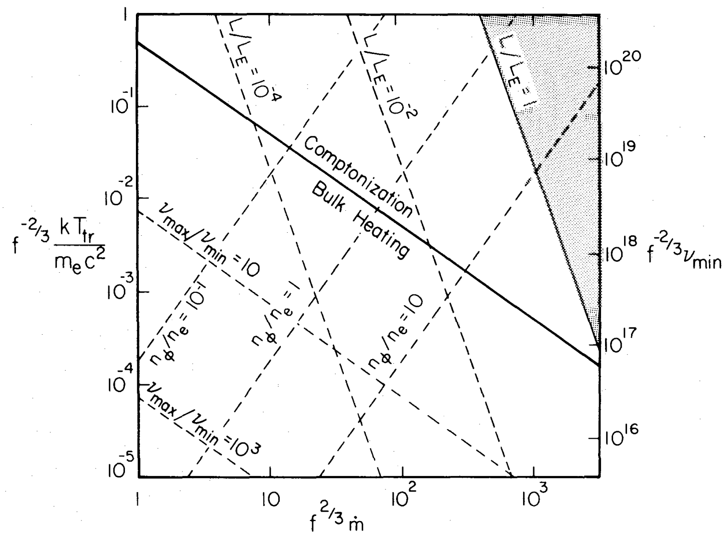

Figure 5. Properties of the radiation field as a function of the temperature at the trapping radius T_{tr}, the clumpiness factor $f(\equiv \langle n_e^2 \rangle / \langle n_e \rangle^2$, assumed constant), and the accretion rate $\dot{m}(\equiv \dot{M}/\dot{M}_E)$. Above the bold line, Comptonization dominates the radiation field, while, below it, bulk heating and bremsstrahlung cooling dominate the radiation field. The dashed curves correspond to different values of L/L_E, the photon-to-electron ratio, n_ϕ/n_e, and the extent of the high-frequency power-law, with ν_{min} (the lower cut-off of the power-law) given on the right-hand ordinate. In the shaded region, radiation pressure influences the dynamics of the flow and our calculations are no longer valid.

An additional source of new photons (with $\nu \sim kT/h$) which we have ignored is double Compton-scattering. This process is more important at r_{tr} than bremsstrahlung when

$$\left(\frac{kT_{tr}}{m_e c^2}\right) > \left(\frac{\pi}{8}\right)^{1/2} \left[\alpha \dot{m} \ln \left(\frac{kT_{tr}}{h\nu_{DC}}\right)\right]^{-1/2}, \tag{48}$$

where

$$\nu_{DC} \simeq \frac{m_e c^2}{h} \left[\frac{\alpha}{4\pi^2} n_\phi \left(\frac{h}{m_e c}\right)^3\right]^{1/2} \tag{49}$$

(Lightman 1980; Thorne 1981).

For conditions of interest here, free–free absorption is always unimportant for photons of frequency $\nu \gtrsim \nu_{min}$.

3.4 PREHEATING

We have not yet demonstrated that a steady supersonic flow of the type that we have described in Section 3.2 can be established. If the flow is cool and subsonic at large distances, then X-ray preheating may limit steady flows to $l \ll 1$ (Ostriker *et al.* 1976). This problem has been analysed by Cowie, Ostriker & Stark (1979) and Bisnovatyi-Kogan & Blinnikov (1980), reaching conflicting results. However, as bound–free and bound–bound processes have not yet been included in a fully self-consistent manner, this matter must still be regarded as open. In particular, comparing Compton heating and free–free cooling rates is not likely to provide much insight into the thermal structure of the flow for temperatures in the range $\sim 10^4$–10^6 K. A detailed consideration of the thermal balance in an X-ray illuminated flow is outside the scope of this paper. Such a study has been done by Hatchett, Buff & McCray (1976) for stationary gas clouds, and, to the extent that their results are applicable to our problem, we can use them to estimate the sonic temperature T_s. If local balance between X-ray heating and radiative cooling is obtained at the sonic radius, then

$$T_s = (6.4 \times 10^5\ \mathrm{K}) \left(\frac{l}{10^{-2}\dot{m}}\right)^{-2} \left(\frac{\Gamma}{10^3}\right)^2, \tag{50}$$

where

$$\Gamma \equiv \frac{L_X}{n_e(r) r^2} \tag{51}$$

is the usual photoionization parameter, with L_X the incident X-ray luminosity. Hatchett *et al.* (1976) have determined $T(\Gamma)$ for the case of a 10-keV thermal bremsstrahlung ionizing spectrum. The intersection of equation (50) with their $T(\Gamma)$ curve gives $T_s(l/\dot{m})$. For $(l/\dot{m}) = 10^{-3}$, $T_s \sim 3 \times 10^4$ K, while for $(l/\dot{m}) = 10^{-2}$, $T_s \sim 4 \times 10^5$ K. At larger values of $(l/\dot{m})$, T_s approaches the Compton temperature ($h\langle\nu\rangle/4k \sim 3 \times 10^7$ K).

Even if preheating does inhibit the formation of a conventional sonic point, supercritical, supersonic flows may still be established For example, preheating may result in the flows becoming two-phase when they are transonic (McCray 1979). Alternatively, the gas may be supplied within the conventional sonic point. In the context of a quasar model, if gas is supplied by the collisions or tidal disruption of stars, the gas will probably expand and cool so that its sound speed is less than the escape velocity, at the point of injection.

4 Non-thermal emission

Unless there is a large clumping factor, free–free processes do not lead to a high radiative efficiency. Several authors (e.g. Schvartsman 1971; Novikov & Thorne 1973; Ipser & Price 1977) have pointed out that synchrotron radiation can give a higher photon yield. We assume that there is a radial (split-monopole) magnetic field supported by toroidal currents in a massive equatorial disc in Keplerian orbit about a massive black hole (*cf.* Blandford & Znajek 1977; Bisnovatyi-Kogan & Ruzmaikin 1976). Within some radius beyond the photosphere, the magnetic stresses $\sim B^2/4\pi \propto r^{-4}$ exceed the Reynolds' stress $\rho v^2 \propto r^{-5/2}$ and radial flow is assured. We also assume that relativistic electrons can be accelerated continuously within the flow in the vicinity of the trapping radius. The free energy for this is most likely to be extracted electromagnetically from the hole (Blandford & Znajek 1977).

If the electron spectrum $n_\gamma \propto \gamma^{-p}$ extends up to $\gamma \sim \gamma_{\rm crit}$ with $p > 2$ and the ratio of the relativistic particle pressure to thermal pressure is $\delta \lesssim 1$, then it is straightforward to show that

$$l = 2/9\, \delta \gamma_{\rm crit} \left(\frac{v_A}{u}\right)^2 \left(\frac{v_e}{u}\right)^2, \tag{52}$$

where v_e is the electron thermal velocity and v_A is the formal Alfvén speed, $B/(4\pi n_e m_p)^{1/2}$. All quantities are evaluated at the trapping radius. By assumption (v_A/u) and $\gamma_{\rm crit}$ exceed unity, hence the requirement $l \lesssim 1$ ensures that $v_e < u$, unless δ is very small. Thermal Comptonization of the photons is then less important than bulk accretion. (This also guarantees that relativistic inverse-Compton radiation is ignorable.) The condition for neglect of synchrotron self-absorption at the maximum emitted frequency $\nu_{\rm crit} \sim \gamma_{\rm crit}^2 (eB/6\pi m_e c)$ is roughly

$$\nu_{\rm crit} \gtrsim \frac{9\pi}{4} \left(\frac{c}{r_e}\right) l \left(\frac{u}{v_A}\right)^2 \gamma_{\rm crit}^{-5}, \tag{53}$$

where r_e is the classical electron radius. For large enough γ_{crit} (typically $\gamma_{crit} \gtrsim 10$), synchrotron self-absorption is ignorable. Free–free absorption at γ_{crit} will be unimportant provided that

$$\nu_{crit} \gtrsim \frac{1}{4}\left(\frac{3}{\pi^3}\right)^{1/4} \frac{c}{(r_e r_s)^{1/2}} \left(\frac{v_e}{c}\right)^{-3/2} \dot{m}^{1/4} \bar{g}^{1/2}. \tag{54}$$

The relativistic electrons will also radiate bremsstrahlung γ-rays which can escape more readily from the flow than the synchrotron photons due to the reduced opacity at γ-ray energies. (The ratio of the bremmstrahlung to the synchrotron emissivity is $\sim \alpha(m_e/m_p)$ $(c/v_A)^2 (\ln \gamma_{crit}/\gamma_{crit})$ which is generally smaller than unity.) Coulomb losses will also heat the thermal plasma, but this will be inconsequential on the infall time-scale as long as $\delta < \frac{1}{3}\gamma_{crit}/(\dot{m} \ln \Lambda)$, where $\ln \Lambda \sim 30$ is the Coulomb logarithm.

Under the conditions just outlined, the synchrotron process can also provide an efficient source of photons in the vicinity of the trapping surface. For $\nu \lesssim \nu_{crit}$, the spectrum will be determined by the radial variation of δ and γ_{crit}. For $\nu \gtrsim \nu_{crit}$, the spectrum will be a power-law of extent

$$\frac{\nu_{max}}{\nu_{min}} \sim \left(\frac{2}{3}\right) 6^{1/4} \alpha \left(\frac{m_e}{m_p}\right)^{1/2} \left(\frac{c}{v_A}\right)\left(\frac{r_s}{r_e}\right)^{1/2} \dot{m}^{-3/4} \gamma_{crit}^{-2}, \tag{55}$$

with an index $\alpha \sim 2$. Plausible and self-consistent parameters for a $10^8 M_\odot$ black hole in a galactic nucleus are $\gamma_{crit} \sim 30$, $\dot{m} \sim 10$, $v_A \sim 3c$, $\delta \sim 10^{-2}$ and $T \sim 10^7$ K. They produce a power $l \sim 0.3$ with a peak synchrotron frequency $\nu_{crit} \sim 3 \times 10^{13}$ Hz. The high-frequency power-law extends up to $\nu_{max} \sim 3 \times 10^{18}$ Hz.

The synchrotron process is not the only possible source of non-thermal photons in a cold radial flow. For a massive black hole in a galactic nucleus, the magnetic strengths required for a significant *thermal* cyclotron power are implausibly large ($\sim 10^8$ G). However, conditions may be conducive to the operation of a cyclotron maser which would allow the maintenance of a radiation brightness temperature far in excess of the effective electron kinetic temperature. Alternatively, coherent radiation of electron plasma oscillations by streaming electrons could produce a high power at the plasma frequency (~ 1 GHz) (*cf.* Colgate, Colvin & Petschek 1975). In both instances, steep ($\alpha \sim 2$) power-laws extending over many decades of frequency would be expected.

5 Discussion

The aim of this paper has been to discuss bulk acceleration of photons in a spherical inflow and to demonstrate that this process can indeed operate under conditions that are dynamically and radiatively self-consistent. It has not been to produce a serious quasar model and the examples of the previous two sections are not intended as such. Nevertheless, we can comment briefly upon the most likely application of these ideas to massive black-hole models of active galactic nuclei.

First, the spectral index produced for free-fall, $\alpha = 2$, is generally steeper than usually observed for the optical and X-ray continua. Two exceptions are BL Lac objects (e.g. Stein 1978) and the infrared spectra of some quasars (e.g. Soifer 1980); although in the former case the emergent radiation is strongly polarized, whereas in the latter it is time-steady. Neither condition is likely to be satisfied in a spherical inflow. Furthermore, bulk acceleration leads to only a small energy-amplification (typically $\lesssim 1.4$) and most of the emergent luminosity is derived from the emission process which is not at all specific to spherical accretion.

The majority of quasars have unpolarized optical spectra with $0.5 \lesssim \alpha \lesssim 1.5$, that frequently show non-power-law spectral features. Spectra this flat could be produced in the converging flow if the density increased, with decreasing radius, more rapidly than $n_e \propto r^{-3/2}$. This may occur if $l \sim 1$ and the radiation produced in the vicinity of the trapping radius is sufficient to decelerate the flow, or if the gas is channelled by converging magnetic field lines. In particular, for a variation $n_e \propto r^{-3}$ or faster, $\alpha \lesssim 1$, most of the radiated power is derived from the bulk motion, and the associated radiative efficiency is correspondingly high.

If most of the emergent power is derived from other particle heating processes and the gas is in free-fall, then the $\nu^{-1/2}$ spectra predicted for $\nu \lesssim \nu_{\rm min}$ when free–free emission dominates is suggestive of the observed X-ray spectrum of Seyfert galaxies (Holt 1979). Bulk acceleration may act as a secondary particle-acceleration mechanism with the result that the X-ray spectra of Seyferts and quasars would show high-frequency power-law tails with $\alpha \sim 2$. Once a high-frequency break in the X-ray spectrum of active galactic nuclei has been observed, such an effect could be looked for, although the extent of the power-law tail is unlikely to be large.

We have not given a self-consistent treatment of the transfer of radiation beyond the Compton photosphere. Under some circumstances, the gas will be able to cool so that the photoelectric opacity is significant. (This itself must be determined self-consistently under the conditions of high radiation-density present in our model, *cf.* Paper II.) In principle, such effects could be related to detailed observed spectra. They may also have dynamical consequences.

The assumption of spherical symmetry is necessary to produce a smoothly converging flow of the type discussed (*cf.* Fabian *et al.* 1976; McCray 1979). The best observational argument for spherical symmetry would appear to be the low optical polarization measured in most quasars; either the geometry is quasi-spherical, or electron scattering does not contribute significantly to the effective photospheric opacity in the regions where the optical continuum is produced. However, it seems unlikely that the accreting gas would have so little angular momentum that centrifugal forces do not become important. One way of transporting angular momentum radially outwards is if the flow is strongly magnetized as described in Section 4.

The conditions under which this mechanism is of greatest potential interest are those when l and $\dot{m}$ do not differ much from unity and the radiative efficiency is reasonably high. However, under these conditions relativistic effects (both special and general) that are omitted from our Newtonian calculations might be expected to play a role. Furthermore, the mean number of scatterings a photon undergoes in escaping from the hole is $\sim \dot{m}^2$ and the diffusion approximation must be called into question.

It is important to realize that the physics we have described in this paper does not have an associated mass scale as long as free–free absorption is unimportant. For a given $\dot{m}$ and l, t_i, t_b and t_c are all proportional to r_s. Therefore, these calculations are equally applicable to discussions of compact objects of stellar mass in binary X-ray sources.

Finally, we remark that a mechanism directly analogous to this could in principle occur with cosmic rays replacing photons (e.g. in accretion on to a neutron star). This has been discussed independently by Cowsik & Lee (1981). It is straightforward to show, however, that, under conditions believed to be occurring within the Galaxy, this is unlikely to be significant for the acceleration of galactic cosmic rays.

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